## APPARATUS

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Research is reported on the mixing of two gas fluxes in a layer of granular material.

Interacting gas flows injected at different points are involved in many engineering equipments containing stationary granular beds such as shaft lime furnaces and direct-reduction ones. Those gases usually react with the production of heat and additional gas, which influences the heat transfer and the distribution and thus the throughput and ultimately determines the product quality. To control the process, one needs to know the reaction zone position and the concentration distribution and relation to injection conditions.

There are several papers [1-3] on the gas distributions and interactions in shaft furnaces with intersecting flows of gases; the gas flows have been visualized [1] by means of the coloring of chalk wetted with a starch solution. There are three regions: the one covered by the lateral gas flow, the mixing one, and the region filled with gas passing downwards. Sampling in the bed has given [2, 3] the concentrations, from which it has been concluded by analogy with [1] that there are three zones.

All the available evidence on the concentration patterns has been obtained by experiment, buta shortcoming of the [1-3] approaches is that the approximation formulas for the patterns were derived for single equipment configurations and restricted parameter ranges: particle diameter, number of jets, Re, and so on. The feature restricts the application, and it is necessary to employ more fundamental physical concepts in order to calculate the pattens, where the latest computer methods could be applied to any geometry and injection conditions.

Here we use a model in calculating the patterns for interacting flows in an apparatus of any form; we also perform tests to display the flows and evaluate the concentration distributions. The model is confirmed by the measurements.

The gas flows are displayed by processing the granular material in a dye solution, which causes it to become colored on interaction with an indicator gas, for which we used ammonia or hydrochloric acid. This produces colored flow regions. We used polystyrene granules in a planar bunker having transparent walls, with air supplied together with the indicator gases from the side and bottom.

Figure 1 shows that the apparatus includes the planar model 1, dimensions $342 \times 820 \mathrm{~mm}$, which contained the fine-grain polystyrene 2 , which had been treated with an alcoholic dye solution containing additives. At the bottom, there was a distribution grid 3, which received gas from the source 4, which was filled with concentrated hydrochloric acid. Immersed in the acid was the perforated tube 5, through which compressed air entered via the control valve 6 and flowmeter 7.

An analogous just was provided by the side hole 8 , with compressed air supplied through the valve 9 , flowmeter 10, perforated tube 11 , and tank 12 filled with ammonia.

The flows were displayed as follows. The compressed air passed to the valves 6 and 9 , with monitoring by the flowmeters 7 and 10 , which were used to set the required rates; tubes

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Fig. 1. The apparatus: 1) planar bunker; 2) polystyrene granules; 3) distribution grid; 4) tank containing hydrochloric acid; 5) perforated tube; 6) control valve; 7) flowmeter; 8) lateral hole; 9) valve; 10) flowmeter; 11) perforated tube; 12) tank containing ammonia; 13 and 14) infiltration regions; 15) flow separation line.


Fig. 2. Results: 1) flow separation line; 2) boundaries to mixing one.

5 and 11 bubbled the air through the acid and ammonium whose vapors entered the grid 5 and side hole 8, and thus entered the granular material 2 , where the reactions with the dye indicated the flow regions 13 and 14 , which became colored red and blue. The ammonium and acid reacted together to give a white band against the colored background, which indicated the mixing zone. Figure 2 shows the results.

Various flow ratios were used and various Re corresponding to the laminar, transitional, and turbulent modes; the mixing zone width was dependent on $R e$ in the transition state, while Fig. 2 shows the turbulent state (Re $\approx 1000$ ).

The flow pattern agrees qualitatively with the [1-3] results: there is a zone filled by one component, a narrow mixing zone, and a region filled by the other component. Then arrow indicates the lateral injection point. The point of junction between the flows at the wall lies below the latter.

The experiment was also performed in other ways, in particular, with air supply at the bottom without the tank 4 through the grid 3, while at the side, we injected the indicator gas, which was indicated by the color. Also, after the bed has been treated with one of the indicator gases, one can react it with the other vapor. One can interchange the gases supplied with the air almost. indefinitely. i.e., HCl can be followed by $\mathrm{NH}_{3}$ (one gets a blue color against a red background) and then HCl can be supplied and so on. In that case, one obtains the flow separation line and the propagation front, but the mixing zone is not displayed very clearly. That method was used without a bottom flow, and it was found that the jet propagated in the bed as concentric semicircles, so the flow is similar to that from a point source [4]. We conclude that the surface of the apparatus influences the flow lines only in a narrow region near the wall, i.e., the gas jet fills the bed in the form of semicircles up to the point where the jet interacts directly with the wall.

The model is based on the motion for high Re being described by

$$
\begin{gather*}
-\frac{\partial p}{\partial x}=k_{1} u+k_{2} u\left(u^{2}+v^{2}\right)^{1 / 2} \\
-\frac{\partial p}{\partial y}=k_{1} v+k_{2} v\left(u^{2}+v^{2}\right)^{1 / 2}, \quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}
\end{gather*}
$$

Nevertheless, many researchers such as [5] have used a potential model for gas flow in calculating the velocity patterns and flowlines for equipment containing homogeneous granular beds.

Vortex-free motion was assumed, so one only has to solve the Laplace equation with appropriate boundary conditions; it has been demonstrated [5] that a potential model is applicable, in particular from a comparison of calculations from the complete equations and the linear ones, which showed that vortex motion has effects only near the injection point.

A simple model for a planar equipment shows that the main difference between potential and nonpotential flows makes itself felt only directly by the inlet.

The system here is obtained by eliminating the pressure $p$ from (1) and introducing the flow function $\psi$ :

$$
\begin{gather*}
k_{1}\left(\frac{\partial^{2} \psi}{\partial x^{2}}+\frac{\partial^{2} \psi}{\partial y^{2}}\right)+k_{2}\left(\left(\frac{\partial \psi}{\partial x}\right)^{2}+\left(\frac{\partial \psi}{\partial y}\right)^{2}\right)^{\frac{1}{2}}\left(\frac{\partial^{2} \psi}{\partial x^{2}}+\frac{\partial^{2} \psi}{\partial y^{2}}\right)+ \\
+k_{2}\left(\left(\frac{\partial \psi}{\partial x}\right)^{2}+\left(\frac{\partial \Psi}{\partial y}\right)^{2}\right)^{-\frac{1}{2}}\left(\left(\frac{\partial \psi}{\partial y}\right)^{2} \frac{\partial^{2} \psi}{\partial y^{2}}+\right.  \tag{2}\\
\left.+2 \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} \frac{\partial^{2} \psi}{\partial x \partial y}+\left(\frac{\partial \psi}{\partial x}\right)^{2} \frac{\partial^{2} \psi}{\partial x^{2}}\right)=0
\end{gather*}
$$

Let the injection conditions be such that the axial velocity $u$ is larger than the transverse velocity $v$; then (1) may be solved as a small-parameter expansion, where we take only two approximations:

$$
\begin{equation*}
\psi=\psi_{0}+\lambda \psi_{1}+\ldots, \tag{3}
\end{equation*}
$$

in which $\lambda$, the small parameter, characterizes the inhomogeneity in the longitudinal gas speed profile.

The zeroth approximation corresponds to a flow with no transverse component; the first characterizes the deformation in the initial profile, where

$$
\begin{equation*}
\left.\psi_{1}\right|_{y=0}=0 ; \quad \psi_{1} / y=R=0 . \tag{4}
\end{equation*}
$$

The boundary conditions for (3) correspond to conservation of the injected mass; the solution to the linearized equation ( $\psi_{0}=u_{a v} y$ ):

$$
\begin{equation*}
\psi_{1}=\Sigma A_{k} \sin \left(\gamma_{h} \frac{y}{R}\right) \mathrm{e}^{-\sqrt{2} \gamma_{k} \frac{x}{R}}, \tag{5}
\end{equation*}
$$

which implies that $y_{k}=\pi_{k}$ in order to meet the homogeneous conditions at the axis and wall.
The exponent increases rapidly with the number in the series, so the specific contribution from the first term increases rapidly with $x$, and one can evaluate the solution largely from it: the inhomogeneity in the velocity profile decreases as $A^{*}=A_{0} \mathrm{e}^{-\sqrt{2 \pi} x / R}$, and for potential flow, one can similarly derive $A^{*}=A_{0} e^{-\pi x / R}$, so at sufficient distances from the inlet, the two models give similar results.

The flow mixing with homogeneous packing can be considered from the potential model, because the mixing usually occurs in the middle far from the inlet and the error in determing the velocities on account of the nonpotential flow is minor.

The mixing is described as follows: at the middle of the mixing zone, we draw the surface $f(x, y, z)$ and neglect the zone to consider the flow as the interaction between two emissible liquids; one finds an analytic expression for $f(x, y, z)$ and solves for diffusion in the mixing layer. This is a classical scheme for gas dynamics. We consider it convenient and promising for infiltration flows in fixed-bed equipment, particularly shaft ovens.

Laplace's equation gives the distribution in a uniform bed without flow mixing on the assumption that the gases are similar in density; the boundary conditions follow from the impermeability of the wall, the injection conditions, and the constant pressure at the upper boundary:

$$
\begin{equation*}
\Delta \varphi=0, \quad \frac{\partial \varphi}{\partial n}=0 ; \quad M \notin S ; \quad \frac{\partial \varphi}{\partial n}=V_{0} ; \quad M \in S . \tag{6}
\end{equation*}
$$

Here $M$ is a point on the side surface, $V_{0}$ the injection speed, and $S$ the injection surface.
The flow separation surface equation is derived from the condition for the normal component of the gas velocity there being zero, i.e., the no-flow condition

$$
\begin{equation*}
\frac{\partial \varphi}{\partial x} f_{x}+\frac{\partial \varphi}{\partial y} f_{y}+\frac{\partial \varphi}{\partial z} f_{z}=0 . \tag{7}
\end{equation*}
$$

That surface should arise at the wall and be the one of the surfaces that satisfies (7).
We consider the solution for the planar flow used in the measurements. Then (7) coincides with the equation for the current line separating the flow from the source and the incident flow region. The measurements and calculations [4] show that for sufficiently small ratios of the injection hole diameter to the equipment diameter, the flow can be considered as from a point source. The complex-variable method [6] gives the equation for the set of current lines:

$$
\begin{equation*}
\psi=V_{\infty} y+\frac{Q}{\pi} \operatorname{arctg} \frac{y}{x-x_{0}} . \tag{8}
\end{equation*}
$$

Let $x^{*}$ be the coordinate for the point of rotation from the surface of the equipment for the vertical gas flow entiring from below, as the rotation occurs because of the interaction between the two oppositely moving flows, and the velocity at that point becomes zero. Then

$$
\begin{equation*}
V_{\infty}=\frac{Q}{\pi\left(x_{0}-x^{*}\right)} . \tag{9}
\end{equation*}
$$

That point becomes the initial point for the flow separation line; we substitute (5) into (8) to get the equation for that line as

$$
\begin{equation*}
x=x_{0}-\frac{y}{\operatorname{tg} \frac{y}{x_{0}-x^{*}}} . \tag{10}
\end{equation*}
$$

We use the equation for the boundary diffusion layer referred to that surface to describe the mixing zone:

$$
\begin{equation*}
v_{n} \frac{\partial c}{\partial n}+v_{\tau} \frac{\partial c}{\partial \tau}+v_{b} \frac{\partial c}{\partial b}=D \frac{\partial^{2} c}{\partial n^{2}} . \tag{11}
\end{equation*}
$$

Here $\mathrm{v}_{\mathrm{T}}$ is the projection of the velocity on the tangent to the line of intersection between the surface and the vertical section, while $v_{n}$ is that on the normal and $v_{b}$ that on the binormal.

To simplify solving (11), we assume that the two gases move at the same speed along the separation line, which is equal to the mean speed, while the transverse velocity $v^{0}$ can be neglected. We neglect the curvature in the separation line and consider the equation along the straight line to get

$$
\begin{equation*}
u^{0} \frac{\partial c}{\partial x^{0}}=D \frac{\partial^{2} c}{\partial y^{0^{2}}} . \tag{12}
\end{equation*}
$$

The boundary conditions for (12) can be derived from a model for an asymptotic boundary layer, $\tau . \% ;(\infty)=1 ; \quad c(-\infty)=0$; simple steps give the concentration as

$$
c=\frac{1}{2}\left(1-\int_{0}^{\beta} \mathrm{e}^{-\frac{1}{4} \boldsymbol{\beta}^{2} \operatorname{ReSm}} d \boldsymbol{\beta}\right) .
$$

The diffusion coefficient is [7]

$$
D=A D_{\mathrm{g}}+B_{0} u^{0} d_{k} .
$$

$B_{0}$ is defined from the [7] data for mixing of parallel-moving flows as 0.2 ; in the turbulent state, the first term here can be neglected, and then $\operatorname{ReSm}=1 / B_{0}$. As an asymptotic model is used for the boundary layer, in which the thickness is a nominal quantity, to derive it, one has to specify the nominal concentration range. We take the mixing zone as the region where $\alpha<c<1-\alpha$, and a comparison was made with experiment for $\alpha=0.1$ (Fig. 2), which corresponds to $10 \%$ of the range and is an accuracy sufficient for practical purposes. The dashed lines in Fig. 2 show the calculated boundary-layer boundaries. The solid line is the boundary between the flows defined by (10).

The good agreement shows that the flow is potential in an isotropic medium (including at high Re corresponding to inetial flow) and this shows that the model is applicable.

## NOTATION

$x$, vertical coordinate axis; $y$, horizontal axis; $u$ and $v$, velocity components corresponding to the x and y axes; $\mathrm{x}^{0}$, coordinate along the separation line; $\mathrm{y}^{0}$, coordinate perpedicular to that line; $u^{0}$ and $\mathrm{v}^{0}$, velocity components corresponding to $\mathrm{x}^{0}$ and $\mathrm{y}^{0}$; p , pressure; $k_{1}$ and $k_{2}$, infiltration coefficients in the Ergan law; $d_{k}$, particle diameter in bed; c, concentration of one component; $D$, diffusion coefficient; $\nu$, kinematic viscosity; $\operatorname{sm}=v / D$, Schmidt number; Re $=d_{k} u / v$, Reynolds number; $X_{0}$, coordinate of injection point; $Q$, gas flow through side hole; $\mathrm{V}_{\infty}$, flow speed in gas from bottom; $\beta=y^{0} V \bar{x}^{0} d_{k}$, dimensionsless coordinate; $R$, apparatus half-width; A dimensionless factor that incorporates the layer structure in molecular diffusion; $D_{g}$, molecular diffusion coefficient; $B_{0}$, dimensionless factor incorporating turbulent mixing rate; $\varphi$, velocity potential; $u_{a v}$, average velocity in apparatus.

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A NONEQUILIBRIUM POLYDISPERSE FLOW IN AN AXISYMMETRIC NOZZLE
WITH CONDENSATE PARTICLE COAGULATION AND BREAKUP
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A kinetic description is given for the flow of a polydisperse medium, which is based on the general theory for nonequilibrium processes; the conversion to generalized transport equations is discussed and the closure of the infinite equation chain. An example is given of numerical calculations on such a flow in a nozzle on the basis of particle coalescence and breakup.

1. Theoretical studies may be made on polydisperse media by means of kinetic, statiscal, or phenomenological methods. In the kinetic description of a system having variable mass $m$, speed $U$, temperature $T$, angular coordinates $\Theta$, angular velocity, $\omega$, angular momentum $M$, interaction force $F$, and other such parameters, one introduces the phase volume dl' and a generalized 14 -dimensional phase space.

$$
d \Gamma=d \mathbf{r} d \mathbf{U} d \Theta d \omega d T d m
$$

with the generalized distribution

$$
f_{i}(t, \mathbf{r}, \mathbf{U}, \boldsymbol{\Theta}, \omega, m, T)
$$

We consider the motion and particle interaction in phase space and get the general kinetic equation there for the distribution:

$$
\begin{equation*}
\frac{\partial f_{i}}{\partial t}+\mathbf{U}_{\nabla \mathrm{r}} f_{i}+\nabla \mathrm{U}\left(\frac{\mathbf{F}_{i}}{m_{i}} f_{i}\right)+\omega_{\nabla \mathrm{V}} f_{i}+\nabla_{\omega}\left(\frac{\mathbf{M}_{i}}{I_{i}} f_{i}\right)+\nabla_{\nabla}\left(\frac{\mathrm{q}}{m_{i} c_{i}} f_{i}\right)+\nabla_{m}\left(\frac{d m_{i}}{d t} f_{i}\right)=J_{i j} . \tag{1}
\end{equation*}
$$

One of the most difficult aspects of kinetic theory is to determine the collision integral $J_{i j}[1-4]$ in (1); if the medium is of sufficiently low density, it can be neglected [5-7]:

$$
\begin{equation*}
J_{i j}=0 \tag{2}
\end{equation*}
$$

Near thermodynamic equilibrium, that integral can sometimes be represented approximately as a relaxation relation

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